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The Riskiness of Risk Models: Assessment of Bankruptcy Risk of Non-Financial Sector of Pakistan

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Abstract

Bankruptcy prediction has long been an important concern for various stakeholders in an increasingly intricated business environment. Using a sample of 3,806 company-year observations of listed non-financial companies of Pakistan during 2005-2015, the paper compares models and identifies an optimal approach in terms of forecasting accuracy for predicting financial distress and bankruptcy. The purpose of the study is to develop a model with relatively high predictability and figure out determinants of bankruptcy. By employing financial ratios, equity market variables and macroeconomic indicators; the hybrid artificial neural network (ANN) validates superior performance as opposed to dynamic panel probit and Merton (1974) models individually. Among financial ratios; quick ratio, cash ratio, current to total asset, quick to total asset, cash flow to short-term debt, gross profit margin, asset turnover, interest to debt, net working capital to net sales, and cash to net sales are crucial in examining firm's financial status. Additionally, money supply, forex reserves, exchange rate, balance of trade, and real GDP growth rate are found statistically meaningful in predicting bankruptcy.

Keywords: *Bankruptcy prediction, credit risk, hybrid-ANN, Dynamic Binary Panel Probit*

JEL classification: *C25, C45, G30, G33*

1. Introduction

Bankruptcy is the legal status of a business entity reflecting a sobering economic reality of corporate or financial institutions which is the likely outcome of financial distress. Given the dynamic nature of the characteristics of financially distressed firms, it is essential for stakeholders and practitioners to examine different aspects of bankruptcy and enhance the performance of existing bankruptcy prediction models.

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Bankruptcy or failure is defined as "*the inability of a firm to pay its financial obligations as they mature*" (Beaver, 1966, p. 71). The financial distress arises when a company has mostly illiquid assets, high fixed costs, or revenue sources that are exposed to the business cycle which ultimately leads toward bankruptcy. Financial distress proxy default risk and the lender would ask for an additional premium to compensate that risk. The other related terms found in the literature include failure, insolvency, and default. We use these terms interchangeably as in previous studies (Altman & Hotchkiss, 2006).

Suboptimal provision of resources because of the faulty prediction of the credit-worthiness of counterparty can incur significant losses. A classic example to this end is the sub-prime mortgage crisis of 2007-08 which bankrupted even venerable banks and companies like Lehman Brothers, Washington Mutual, General Motors, etc. The predictive ability of traditional financial distress prediction appears limited during the period of the Global Financial Crisis of 2007-08 (Ashraf, Felix, & Serrasqueiro, 2019). Bankruptcy prediction has attracted researchers since the work of Beaver (1966) and Altman (1968). In early prediction models, financial statements of the companies are considered as the only source in assessing the financial position of the company. Later studies, however, include equity market characteristics and macroeconomic variables as prospective predictors. Ohlson (1980) uses financial ratios to forecast bankruptcy and also highlights the significance of equity market variables in reviewing financial status. Agarwal and Taffler (2008) argue that stock prices reflect all information in financial statements and contain additional information. Other studies including Tinoco, Holmes, and Wilson (2015), and Tinoco and Wilson (2013) conclude that utilization of financial ratios, stock market variables, and macroeconomic indicators simultaneously can enhance forecasting accuracy. Following previous studies, the current study argues that the financial health of the firm primarily contingent on three sources of risk which firstly include the firm's idiosyncratic characteristics like profitability, liquidity, efficiency, and solvency which can be proxied through financial ratios. Economic conditions and business cycle positioning are also relevant in examining bankruptcy, therefore, macroeconomic variables are employed. Besides, equity market variables are considered forward-looking and reflect investor's perception toward the financial conditions of the firm, therefore, these variables are also expected to enhance the predictive performance of the model.

Bankruptcy risk is highly pronounced in developing countries. It has been argued that sovereign risk is associated with an aggregate default risk of a country's corporate sector (Altman & Rijken, 2011). Pakistan is among those countries that are critical to default risk as international rating agencies like Moody and Fitch have downgraded Pakistan's long-term debt rating from 'stable' to 'negative' outlook in the third quarter of 2018. Persistent economic and political turmoil along with the absence of stringent bankruptcy laws has also heightened the cost of doing business. The huge failure of textile firms in Pakistan in the late 2000s brings concrete evidence. About 0.2 million power looms and at least 40 percent of Pakistan's textile industry have either fled to Bangladesh or shut down their businesses over 2007-2012. Therefore, it is imperative to assess the prevailing financial conditions of the non-financial sector in Pakistan as there is no effective legitimate business protection available. Moreover, no international study particularly focuses on Pakistan that evaluates the causes and effects of the bankruptcies.

A large number of models and corresponding methodologies have evolved for detecting financial distress, however, the problem of developing an optimal approach still stands. The current study is devoted to identifying an optimal model for examining the financial distress of the non-financial sector of Pakistan. These models include dynamic panel probit, hybrid artificial neural network, and Merton's (1974) model which are carefully selected in such a way that each model captures a distinct underlying aspect.

This paper contributes to the existing literature in numerous ways. It firstly focuses on the Pakistani non-financial sector over the period of 2005-2015 and classifies firms between bankrupt versus non-bankrupt by combining finance-based definition along with legal bankruptcy classification. Pakistan has a diversified industrial base that particularly includes textile, food, pharmaceutical, paper and paper board, and other industries. Moreover, the strategic ties between China and Pakistan have touched its peak with a visible economic expansion in the wake of the China Pakistan Economic Corridor (CPEC) project. Therefore, insights pertaining to business risk characteristics in Pakistan and the identification of an optimal approach to assess the same is an interest of various stakeholders.

Second, it appears that firms with financial complexities in previous years are more likely to get bankrupt thus autoregressive scheme seems reasonable. Therefore, the dynamic panel probit model of Wooldridge (2005) is employed for the first time in evaluating bankruptcy. Further, data science is an emerging technology and models like Artificial Neural Networks (ANNs) have been widely reported for their appealing accuracy. Unlike traditional statistical methods such as Multiple Discriminant Analysis (MDA), ANN is not grounded on stringent statistical assumptions but suffers from overfitting problems when there are more explanatory variables (Khemakhem & Boujelbene, 2018). The integration of dynamic panel probit model and ANN (i.e. hybrid-ANN) controls an issue of overfitting while retaining higher predictability of ANN. Hybrid-ANN is the second model of the study and this study extends the relevant literature by providing hybridization of two approaches for predicting bankruptcy. In addition, the third model (i.e. Merton-model) of the study is a simplified version of Moody's-KMV- a proprietary model of Moody's Analytics- and can be viewed as of practical significance. Many studies (such as Gharghori, Chan & Faff, 2007; Agarwal, & Taffler, 2008; Vassalou & Xing, 2004) argue about the widespread success of Merton model.

Finally, using a panel framework along with three types of variables i.e. financial ratios, macroeconomic variables, and equity market variables simultaneously (similar to Tinoco & Wilson, 2013) add value to the existing literature.

The results of the study broaden the existing literature in the following ways. Financial ratios and macroeconomic variables are found statistically significant while equity market variables failed to add any value in predicting bankruptcy. The poor predictability of equity market variables raises concerns over stock market efficiency and brings evidence of mispricing of default risk in equity returns. The hybrid ANN model (i.e. a combination of ANN and dynamic panel probit model) attained the highest predictive accuracy for both in-sample and out-of-sample forecasts. Both models are complementary as a dynamic panel model provides interpretability of the coefficient while hybrid ANN enhanced the forecasting accuracy. Moreover, the study provides a material definition of bankruptcy for the non-financial sector of Pakistan.

The rest of the study is structured as follows. Section 2 provides a review of existing literature. Section 3 is the research design. Section 4 describes the empirical results. Section 5 exhibits model diagnostic and, finally, Section 6 presents the conclusion.

2. Review of Literature

Most of the previous studies employ a legal definition of bankruptcy and are modelled through binary choice models based on a distinctive population of bankrupt and non-bankrupt companies. However, a legal definition has some limitations. For instance, the bankruptcy filing does not guarantee insolvency, it may be seeking temporary protection and vice versa may also be true (Haber, 2011). Moreover, a legal filling can be a lengthy and costly process and may not be representative of a true economic event of failure. In the context of Pakistan, a plethora of ineffective and flawed bankruptcy laws are prevailing while businesses are at their own risk. Hasanain and Shah (2012) examine the Corporate Rehabilitation Act (CRA) (Securities and Exchange Commission of Pakistan, 2009d) and found it defective as well as more debtors' friendly rather than optimal. A recent study of Luqman, Hassan, Tabasum, Khakwani, and Irshad (2018) defines financial distress as the situation when a firm generates negative income in the previous five years consistently for non-financial firms of Pakistan. Their study ignores legal classification, moreover, employed definition narrowed down a very small sample. The current study, therefore, uses the definition for quoted companies in Pakistan by combining finance-based definitions along with legal bankruptcy classification. The utility of finance-based definition has been marked significant in the academic literature (Pindado, Rodrigues, & de la Torre, 2008; Tinoco et al., 2015) and supported by the fact that financial distress is more accurately and timely pronounced in financial statements before legal proceedings. The study contributes to the existing literature by the integration of legal and finance-based definition in contrast to individual independent definitions utilized in previous studies.

The lack of theoretical underpinning for bankruptcy has limited the development of a sound scientific approach to bankruptcy prediction while a wide range of empirical frameworks is available. The extensive bankruptcy literature encompasses a variety of methodologies ranging from statistical to data science models. The modelling of financial distress for quoted companies is dated back to two centuries (Altman & Hotchkiss, 2006). Empirical literature mainly emerges after the seminal work of Beaver (1966) followed by Altman (1968). Beaver (1966) studies the predictive abilities of financial statements by using univariate analysis. Altman's (1968) work employed multiple ratios simultaneously with the help of Multiple Discriminant Analysis (MDA). The conditional logit, firstly employed by Ohlson (1980) to generate the probability of default followed by many others (such as Campbell, Hilscher, & Szilagyi, 2008). Shumway (2001) introduces the simple hazard model (similar to multi-period logit) and finds significant improvement in accuracy employing dynamic version. Rashid and Abbas (2011) construct Z-score for Pakistan by modelling financial ratios via MDA. Khan (2018) reveals that the logit model outperforms discriminant analysis in analyzing bankruptcy for the financial sector of Pakistan.

Data science approaches (such as ANN, support vector machines, decision trees, and others) have emerged as a new challenger for traditional statistical models since the 1990s. Altman, Marco, and Varetto (1994) compare linear discriminant analysis (LDA) with ANN. They accept the high predictive ability of ANN but cautious about the architecture of the network. Other studies focus on hybrid-ANN models, such as Lee, Han, and Kwon (1996) hybridize ANN with statistical models like MDA, decision tree algorithm and self-organizing feature map (SOFM) model. Moreover, Jo and Han (1996) integrate ANN by using probabilities of default from different models and utilize them as an input layer in ANN. A recent study of Alaka, Oyedele, Owolabi, Kumar, Ajayi, Akinade, and Bilal (2018) review statistical and artificial intelligence tools in an attempt to identify the model with superior performance. It is argued that hybrid models tend to outperform individual predictive approaches. Beaver, Cascino, Correai, and McNichols (2019) employ a two-pronged methodology by first estimating the discrete hazard model and a non-parametric approach (i.e. Classification and Regression Tree) in the second stage. An interesting study of Mai, Tian, Lee, and Ma (2019) apply deep learning models (i.e. average embedding model and convolutional neural network) for predicting bankruptcy which uses textual data (from Management Discussion and Analysis (MD&A)) in conjunction with accounting and market-based variables.

In contrast to purely empirical methodologies, modern structural default risk models are backed by theory. It explains the likelihood of corporate failure by equity market information solely. One of the popular models in this context stems from Black and Scholes (1973), and Merton's (1974) working on option pricing - recognizes as Merton's model. Various studies like Gharghori et al. (2007), Agarwal and Taffler (2008), Bharath and Shumway (2008), and Vassalou and Xing (2004) utilize Merton's model for finding probabilities of default. Previous studies also tested modelling performance based on types of variables. The relative contribution of three types of variables (i.e. market variables, macroeconomic indicators and financial ratios) is more generally tested in many studies. These studies (Tinoco et al., 2015; Pindado et al., 2008) have concluded a higher predictive ability than those employing financial variables alone. Beaver et al. (2019) use financial ratios, macroeconomic variables, and country-specific variables for group affiliated firms.

Previous studies have not tested dynamic panel probit and hybrid empirical methodologies in the context of Pakistan. Moreover, existing bankruptcy studies have also not included macroeconomic and market-based variables simultaneously along with financial ratios to figure out determinants of bankruptcy in Pakistan. This study aims to fill this gap by testing competitive modelling approaches and identifying statistically meaningful determinants of bankruptcy for the non-financial sector of Pakistan.

3. Data and Methodology

This section aims to view the dataset, the definition of dependent variables, independent variables, and models to be employed.

3.1. Dataset

The data taken for the study is comprised of 346 non-financial publicly quoted companies for the period over 2005-2015. The available financial ratios are taken from Financial Statement Analysis of publicly traded non-financial companies- published by the State Bank of Pakistan. The macroeconomic variables are obtained from Datastream and the stock market-based information is taken from Bloomberg.

The existence of extreme values of variables can potentially distort the results.

This study winsorises[4] all variables/ratios (except macroeconomic) at a 5 % level across all firms. That is, the time series of each firm is replaced by the lowest (highest) 5 % with the 0.05 (0.95) quantile on each variable similar to Bauer (2012). However, the current study winsorises time series of each firm using the pooled data set. There is no further data manipulation. Other studies like Tinoco and Wilson (2013) used '*tangent-hyperbolic*' transformation while winsorise at 90%.

3.1.1. Definition of the dependent variable

This section discusses the definition of the dependent variable. The study combines legal bankruptcy classification with finance-based definition by employing the following conditions:

1. Companies that are deemed to be defaulted in 2015 or later. The intuition is that the companies that are defaulted in 2015, 2016, or 2017, must have financial distress in the previous years[5].

2. Following Pindado et al. (2008), the firm that has negative earnings before income, taxes, debt, and amortization (EBITDA) in the previous two years.

Regarding the first condition, a list of 101 companies from Pakistan Stock exchange is used that includes companies suspended over the period 2015-2017. The first 56 companies are selected from the list as the available data of these companies matches our sample period i.e. 2005-2015. However, remaining companies do not have entire data available as they are defaulted in early years (i.e. before 2015) thus no longer recorded by PSX or SBP. For these 56 companies, a dependent variable for the previous four years ('1' stands for financially distressed and '0' otherwise).

A firm may file bankruptcy for arbitrary protection from their lenders (Haber, 2011). Similarly, failure to the payment of an outstanding annual listing fee is the major reason for companies being suspended by PSX. It is also likely the case that the firm deliberately wants to be delisted or opportunity cost of the annual listing fee is high and so on. To address such problems, the current study uses finance-based definition along with formal bankruptcy as described above. A combination of the aforementioned conditions offers a definition that is employed for the first time for bankruptcy classification in Pakistan.

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4 The winsorisation is used to control outliers at the cost of variability of data. For example, a typical 90% winsorisation set all data below the 5th percentile to the value of the 5th percentile. Similarly, it sets all the data above the 95th percentile.

5 The condition is useful for recent cases only and cannot be used for company defaulted, say, in 2011 because the defaulted company does not have data after 2015.

3.1.2. Independent variables

The variables found common in literature are initially taken for the study. The initial list comprises of 61 variables (see Table 9 in Appendix). The selection of variables from the initial list is guided by the two-step procedure:

Step-1: Test of multicollinearity[6] *and variance inflating factor*[7] (VIF)

The study has estimated correlation and tested whether the correlation between the two variables is different from zero. A variable among the correlated variables is dropped based on corresponding VIF.

Step-2: Step-wise logistic regression model

The stepwise selection procedures are widely used to identify the covariates to be included in the model. From the list of 61 variables, 18 variables qualified after passing through the two-step procedure. The list of the final variables is presented in Table 1.

It is expected for market capitalization to total debt ratio to be inversely related to the PDs. Firms with sufficient liquidity positions also impede bankruptcy so the liquidity ratios are likely to be negatively correlated with PDs. Similarly; profitability, efficiency, and activity ratios are also anticipated to be inversely associated with PDs. However, firms with highly leveraged balance sheets are more likely to have defaulted, therefore, both long-term and short-term debt ratios are predicted to be linked directly with PDs.

Moreover, domestic economic circumstances are also crucial in magnifying or contracting risk exposures. Higher GDP growth rate allows businesses to expand thus mitigating risks. Other macroeconomic variables depend upon the nature of the business. For instance, an increase in the exchange rate may jeopardize the survival of the net importer firm while pacifying the financial conditions of the net exporter company. Which effect dominates? It can be found empirically.

Figure 1 depicts the frequency of bankruptcy (classified on the basis of two definitions above i.e. those which fulfills at least one of the criteria stated above) against years included in the sample.

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6 Multicollinearity is the state of intercorrelations among the predictors. It refers that the predictors (found correlated) are explaining similar variations in dependent variables.

7 Variance Inflating Factor; $VIF_i = \frac{1}{1-R_i^2}$, *where* Ri^2 *is the coefficient of determination when ith independent variable is regressed over others.*

Table 1: List of Independent Variables

Type	Variables	Symbol
Market Variables		
Market	Market Capitalization to Total Debt	M5
Financial Ratios		
Liquidity	Quick ratio	A2
Liquidity	Cash Ratio	A3
Liquidity	Current to Total Asset Ratio	A9
Liquidity	Quick to Total Asset Ratio	A10
Liquidity	Cashflow to Short-term Debt Ratio	A12
Leverage	Short-term Debt to Total Debt Ratio	A19
Leverage	Long-term Debt to Total Asset Ratio	A20
Profitability	Gross Profit Margin	A32
Efficiency	Asset Turnover Ratio	A34
Efficiency	Interest Expense to Debt Ratio	A36
Activity	Net Working Capital to Net Sales Ratio	A38
Activity	Cash to Net Sales Ratio	A43
Macroeconomic Variables		
Macroeconomic	GDP Growth	E2
Macroeconomic	Money Supply (M2)	E3
Macroeconomic	Exchange Rate	E6
Macroeconomic	Forex Reserves	E7
Macroeconomic	Balance of Trade	E9
Macroeconomic	Trade Openness	E10

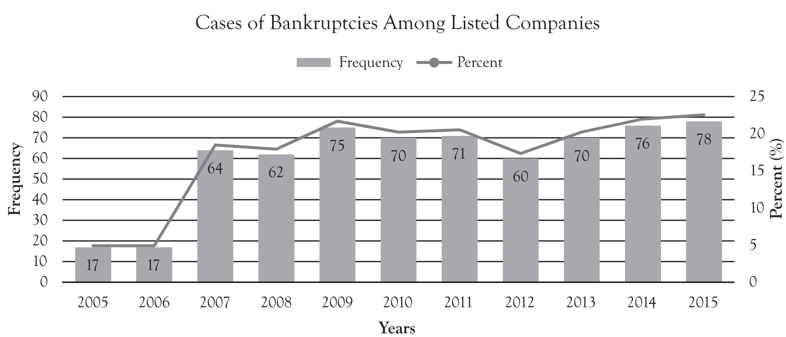

Figure 1: Frequency and Percentage of Bankruptcy Cases Over 2005-2015.

Table 2: Descriptive Statistics

	Non-bankrupt firms		Bankrupt firms		Overall Sample	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
Market Cap. to Total Debt	18.75	98.18	56.88	356.17	25.36	173.62
Quick Ratio	1.44	11.48	12.74	77.68	3.40	34.24
Cash Ratio	0.42	3.02	0.25	3.19	0.39	3.05
Current to Total Asset Ratio	0.48	0.21	0.34	0.26	0.46	0.23
Quick to Total Asset Ratio	0.31	0.34	0.85	2.10	0.41	0.95
Cashflow to Short-term Debt	0.06	0.47	0.75	22.03	0.18	9.18
Short-term Debt to Total Debt	0.75	0.20	0.67	0.30	0.74	0.23
Long-term Debt to Total Asset	0.16	0.17	0.40	0.86	0.20	0.40
Gross Profit Margin	0.13	0.51	-0.48	5.77	0.02	2.46
Asset Turnover Ratio	1.22	0.81	0.57	0.75	1.10	0.84
Interest Expense to Debt	0.04	0.45	0.02	0.21	0.04	0.42
Net Working Capital to Net Sales	0.26	14.44	-6.19	65.91	-0.86	30.51
Cash to Net Sales Ratio	0.05	0.26	0.16	2.14	0.07	0.92
Industries dummies						
1. Textile	0.38		0.60		0.46	
2. Food	0.14		0.08		0.13	
3. Chemicals, Chemical Products and Pharmaceuticals	0.10		0.05		0.10	
4.Other Non-Metallic Mineral Products	0.15		0.12		0.14	
5. Motor Vehicles	0.06		0.01		0.05	
6. Fuel & Energy	0.07		0.06		0.07	
7. Information & Communication Services, other services	0.05		0.06		0.05	
8. Paper, Paperboard & Products	0.03		0.60		0.46	
9. Electrical Machinery & Apparatus	0.02		0.08		0.13	

3.2. Methodology[8]

In this paper, we employ three empirical modelling approaches in such a way

8 See Table 8 in the appendix for brief review of methodologies for selected bankruptcy prediction studies.

that each model captures distinct aspects. First, dynamic panel probit of Wooldridge (2005) is selected as it accommodates the dynamic nature of the problem and provides useful interpretation by average partial effect estimates. To the best of our knowledge, no previous study has employed the dynamic panel probit model for predicting bankruptcy, therefore, it serves as a contribution of the current study.

Other frequently used models in the literature are employed here i.e. hybrid-ANN model and Merton's model. Former is well-known for greater predictability while later is renowned for its roots in option pricing theory.

3.2.1. Dynamic panel model

The presence of the lagged dependent variable leads to the violation of strict exogeneity of the regressor resulting endogeneity bias that leads to inconsistent estimates (Ullah, Akhtar & Zaefarian, 2018). Arellano and Bond (1991) provide an estimation procedure for the dynamic panel model. Wooldridge (2005), however, identified the approach to model dynamic panel for the limited dependent variable by modeling the distribution of the unobserved effect conditional on the initial value and any exogenous explanatory variables. The advantage of the approach is that strictly exogenous variables along with lagged dependent variable can be easily incorporated as well as it mitigates endogeneity bias. Wooldridge (2005) has extended the Chamberlain's (1980) correlated random-effects approach by specifying auxiliary density as $f\,(c_i \mid x_{it}, Y_{1i})$. The rationale to include dynamic version is the intuition that firms that get bankrupt, bearing hard times in the preceding years, therefore, the autoregressive scheme is reasonable. Literature also evident studies that emphasize the dynamic nature of the bankruptcy (Campbell et al., 2008; Shumway, 2001). The current study uses Wooldridge's (2005) approach for estimating parameters presented in equation (1) and (2). The dynamic model for the bankruptcy prediction employed in this study takes the following form:

$$Y_{it}^{*} = \beta Y_{i(t-1)}^{*} + \gamma' X_{it} + \varepsilon_{it} \qquad (1)$$

$$Y_{i0}^{*} = \gamma' X_{i0} + \varepsilon_{i0}$$

$$\varepsilon_{it} = c_i + u_{it}$$

where Y_{i0}^{*} are the initial values; $Y_{it} = 1\ (Y_{it}^{*} > 0) and Y_i 0 = 1((Y_{i0}^{*} > 0),$

Wooldridge (2005) suggests modeling the distribution of $Y_{it}{}^{*}$ given $Y_{i0}{}^{*}$ and to use conditional maximum likelihood (ML) estimator. It assumes:

$$P(Y_{it}|X_t, c_i, Y_{it}) = \phi(\beta Y_{i(t-1)}^{*} + \gamma' X_{it}\rho_o + \rho_1 Y_{i0} + \rho_2 \overline{X}_i + a_i)$$

$$c_i = \rho_o + \rho_1 Y_{i0} + \rho_2 \overline{X}_i + a_i \qquad (2)$$

where $\phi(.)$ is the normal cumulative distribution function (cdf), X_{it}, is a vector of explanatory variables including financial ratios, macroeconomic and market-based variables respectively; a_i is unobserved individual-effects and is assumed as $a_i \mid \overline{X}_i, Y_{1i} \sim$ i.i.d. N (0, σ^2_a). are the within means vector of the time-varying explanatory variables i.e. $\overline{X}_i = \frac{1}{T}\sum_{t=1}^{T} X_{it}$

3.2.2. Hybrid artificial neural network (ANN)

Research studies on utilizing neural network models for predicting financial distress started in the 1990s[9] and growing continuously. Zhang, Hu, Patuwo, and Indro (1999) show that the neural network outperforms the other prediction models such as a logistic regression model. Jo and Han (1996) use hybrid ANN by combining financial ratios identified by MDA or decision tree and then use them as the input layer neurons in ANN. Their study found that unsupervised MDA- assisted ANN outperforms the others. This study in this respect is analogous to that of Lee et al. (1996) in a way that we identify variables from the statistical model and use them as inputs (neurons) in ANN.

The current study provides a two-way hybridization of ANN. Using three types of variables (i.e. financial ratios, market-based, and macroeconomic variables) simultaneously serves one way (Tinoco et al., 2015). Among these variables, significant variables found in a dynamic panel model is used as input variables for ANN is the second way of hybridization. The hybrid version of ANN appears attractive in addressing much-debated vanishing gradient problem associated with sigmoid function as gradient become increasingly small while dealing with a large number of neurons.

We calculate a weighted sum of the inputs by means of integration function, represented in equation (4), at each hidden node. Each node then uses a sigmoid transfer function to generate an output between 0 and 1, depicted in equation (5). Finally, the sigmoid function bridges the hidden layer and the output layer.

Integration function: $$f_1 = f_1(x) = w_0 + \sum_{i=1}^{k} w_i x_i \qquad (3)$$

Sigmoid function: $$f_2 = f_2(f_1(x)) = \frac{1}{1+e^{-f_1(x)}} \qquad (4)$$

Output: $$\hat{Y} = f_2(f_1(x)) \qquad (5)$$

where $f_1(.)$ is the integration function which is simply the weighted sum of inputs. $f_2(.)$ is the activation function which is nondecreasing, nonlinear, and differentiable.

9 See Hill, Marquez, Connor, and Remus (1994) for the survey of Artificial Neural Network models and its applications.

The cross-entropy (CE) error term is used rather than a sum of squared residual (SSE) as CE is assumed to be better than SSE for binary classification problems. An error-function *E*,

$$E=-\sum_{n=1}^{N}\left[Y_n \log\left(\hat{Y}_n\right)+\left(1-Y_n\right)\log(1-\hat{Y}_n)\right] \tag{6}$$

measures the difference between predicted and actual output, where n=1, 2, ..., N are the observations corresponding to input-output pairs.

Initial weights, employed in equation (3), are drawn from the standard normal distribution and these weights are then iteratively adjusted by using the backpropagation algorithm. Mathematically,

$$w_k^{(t+1)}=w_k^{(t)}-\eta_k^{(t)}.\frac{\partial E^{(t)}}{\partial w_k^{(t)}} \tag{7}$$

where t indexes the iteration steps for *k-th* weight and 'η' is the learning rate and will be increased if the corresponding partial derivative keeps its sign. The partial derivative (gradient) i.e. $\frac{\partial E^{(t)}}{\partial w_k^{(t)}}$ is a sensitivity factor, determining the direction of search in weight space for the weight w_k. This gradient can be expressed as,

$$\frac{\partial E^{(t)}}{\partial w_k^{(t)}}=\frac{\partial E^{(t)}}{\partial f_2^{(t)}}\frac{\partial f_2^{(t)}}{\partial f_1^{(t)}}\frac{\partial f_1^{(t)}}{\partial w_k^{(t)}} \tag{8}$$

The last factor of the right-hand side of equation (8),

$$\frac{\partial f_1^{(t)}}{\partial w_k^{(t)}}=\frac{\partial}{\partial w_k^{(t)}}\left(w_0+\sum_{i=1}^{k}w_i x_i\right)=x_i \tag{9}$$

The derivative of the output neuron with respect to its input is simply the partial derivative of the sigmoid function, it implies

$$\frac{\partial f_2^{(t)}}{\partial f_1^{(t)}}=f_2(.)\{1-f_2(.)\} \tag{10}$$

Finally, the first factor of the right-hand side of equation (8):

$$\frac{\partial E^{(t)}}{\partial f_2^{(t)}}=\frac{\partial E^{(t)}}{\partial \hat{Y}^{(t)}}=Y_n\left(\hat{Y}_n-1\right)+\left(1-Y_n\right)\left(\hat{Y}_n\right) \tag{11}$$

3.2.3. Merton's model

The model belongs to the class of structural models. These types of models have been used extensively by credit rating agencies. These models rely on insights from option pricing theory which states that holding equity is economically comparable to a European call option on the company's asset with the strike price equivalent to the value of debt. According to Merton's model, the firm is treated as defaulted if its market value is less than its debt. The market value of the equity (S_o) is calculated by the Black and Scholes (1973) formula for call options.

$$S_o = V_o \phi(d_1) - B e^{-rT} \phi(d_2) \tag{12}$$

'S_o' is the market capitalization

'V_o' is the firm's assets value (total assets)

'B' is the book value of the firm's liabilities (Long-term Liabilities + Current Liabilities)

'r' is the risk-free interest rate

'T' is the time period for debt maturity

where,

$$d_1 = \frac{\log\left(V_o/B\right) + \left(r + \sigma_v^2/2\right)T}{\sigma_v \sqrt{T}} \tag{13}$$

and, $$d_2 = d_1 - \sigma_v \sqrt{T} \tag{14}$$

where σ_v is the volatility of the asset and $\phi(.)$ is the cumulative distribution function of a Standard Gaussian Distribution. Therefore, the probability of default can be calculated as,

$$P(V_T \le B) = \phi\left(\frac{log(B/V_o) - (r - \sigma_v^2/2)T}{\sigma_v \sqrt{T}}\right) = \phi(-d_2) \tag{15}$$

since V_o and σ_v are not directly observable in any frictionless market, we can find an iterative solution of these quantities with the help of equation (12) together with equation (16).

$$S_o = \frac{\sigma_v}{\sigma_s} \phi(d_1) V_o \tag{16}$$

where σ_s is the instantaneous volatility. It can be calculated by equity returns for each year. Equation (9) provides estimates of the probability of default.

4. Empirical Results

4.1. Results of the dynamic panel model[10]

The results for the dynamic random effect panel probit model are presented in Table 3 which reports the coefficients and average partial effect (APE) of each variable. The coefficients in a non-linear model can only indicate the direction of a relationship whereas APEs reflect the magnitude. The model attained 92.3% of

10 It is to be noted that financial ratios are in absolute form while macroeconomic variables are in natural logarithm form (except for GDP growth rate). Coefficients are, therefore, interpreted accordingly.

the predictive accuracy for in-sample forecasts (Out-of-sample forecast is discussed in Section-5.2). The highly significant coefficient on the lagged firm's status suggests a substantial degree of positive state dependence. The estimate of (i.e. 0.202) indicates that 20% of the total variation is due to individual variation.

Among other explanatory variables, none of the stock market-based variables is found significant. It suggests prevailing market inefficiency as market variables do not contain information regarding the financial distress of the firms and are unable to explain variations in the probability of default. Alternatively, equity market movements are not aligned with the financial conditions of the firm and the stock prices of these firms (with high PD) are not reflecting the true economic reality. Thus, the failure of market variables in explaining the probability of default brings some evidence of default risk anomaly and mispricing of stocks of financially distressed firms. An active investor can earn an abnormal profit by utilizing market inefficiencies. For instance, an investment strategy while exploiting default risk anomaly is to short-sell the stocks of these firms and close the position once firms eventually get bankrupt.

The liquidity (quick ratio, cash ratio, current to total asset, quick to total asset, cash flow to short-term debt), profitability (gross profit margin), efficiency (asset turnover, interest expense to debt), and activity (net working capital to sales) ratios are found significant among financial ratios in explaining the probability of defaults. Except for quick ratio and quick to total asset ratio, coefficients of all other ratios are consistent with the economic rationale. The average increase in the probability of default with one unit increase in the quick ratio is 0.001 while the increase in cash ratio decreases the probability of default by 0.021. Components of both quick and cash ratios are identical except the fact that the quick ratio includes account receivables additionally. Account receivables are generated when a company makes sales on credit which raises counterparty default risk. It implies that the debtor's credit risk is associated with the company's credit risk. Similarly, current to total asset and quick to total asset affects the probability of default in opposite directions. Former decrease the probability, on average, by 0.084 while later increase the probability by 0.04. Analogous interpretation can be made for a positive relationship between quick to total assets and the probability of default. Impact of the liquidity ratios employed in this study akin to that of Deakin (1972). Cashflow to short-term debt (or short-term debt coverage ratio) raises the probability of default by 0.011 units, *ceteris paribus*. A potential explanation is that the companies are devoting most of their operating cash flow in short-term debt servicing thus higher the ratio implies a higher probability of default. Similarly, the contribution of the asset turnover ratio is similar to that of Altman (1968), and Altman, Haldeman, and Narayanan (1977) i.e. the depressing probability of default. Moreover; gross-profit margin, interest expense to debt ratio, net working capital to sales, and cash to net sales are adversely affecting the probability of default and are in line with the economic theory.

Additionally, any increase in money supply, exchange rate, and forex reserves are crucial in enhancing PDs. Money supply, measured by M2, is the sum of currency in circulation, demand deposits by banks, and other deposits. An increase in M2 surges the inflation in the economy which increases the cost of doing business. It is estimated that if the money supply increase by one percent would raise the likelihood of default by 0.04 percentage points, keeping all other variables constant. Moreover, the exchange rate also affects the financial positions of the firms seeking profit from exports or using imported raw materials. Weaker currency expands the cost of raw materials to be imported which complements the profitability of the companies. The coefficient on exchange rate implies that a percent rise in the exchange rate or a domestic currency weakened by one percent will, on average, increase the probability of default by 0.3 percentage points. In addition to this, an increase in forex reserves also smoothens the way towards bankruptcy. In the context of Pakistan, a potential explanation of the positive and statistically significant relationship between forex reserves and probability of default is that forex reserves have mostly increased by foreign borrowing rather than improvements in the balance of trade thus imposing additional risk. So, a percent increase in forex reserves will increase the probability of default by 0.05 percentage points, *ceteris paribus*. Similarly, a percent increase in the trade balance would reduce the likelihood of being default by 0.31 percentage points.

Table 3 also depicts the corresponding results of eight dummy variables to capture idiosyncratic industry effects. It is evident that except for the food industry, other industries possess significantly different default risk characteristics than the textile industry (benchmark category). Fuel industry contributes the highest probability of default (0.025 higher than textiles) among all while paper, paperboard & products industry accounts the lowest individual probability of default (i.e. 0.017 lower than the textile). The potential reason for high default risk in the fuel industry is exposure to both international and domestic economic conditions. International oil prices are relatively volatile and the government generally regulates these prices on the domestic premises. Moreover, the fuel industry is more heavily taxed in the country's jurisdiction than any of the other industries employed in this study. In contrast to this, the paper industry is less likely to get defaulted as most of the inputs of the industry are domestically available and demand (both domestic and foreign) is consistently increasing which remains the industry profitable. Chava and Jarrow (2004) demonstrate the significance of industry effects in assessing the probability of defaults. Furthermore, a dummy is also capturing business cycle variations. It is found that firms during a recession display a higher propensity to get defaulted than during expansion. It is pertinent to note that none of the equity market variables contributes statistically in predicting bankruptcy.

Table 3: Regression Results of Dynamic Panel Probit

Explanatory variables	Coefficients	Average Partial Effects (APEs)
Constant	-19.641 (4.153) *	-
Financially_Distressed0	0.363 (0.171) **	-
Financially_Distressedt-1	1.982 (0.117) *	0.214 (0.214) *
Market Capitalization to Total Debt	-0.002 (0.007)	0.000 (0.001)
Quick Ratio	0.005 (0.002) **	0.001 (0.000) **
Cash Ratio	-0.193 (0.069) *	-0.021 (0.008) *
Current Asset/Total Asset Ratio	-0.776 (0.417) ***	-0.084 (0.047) ***
Quick Asset/Total Asset Ratio	0.370 (0.150) **	0.040 (0.018) **
Cash flow to short-term Debt Ratio	0.102 (0.012) *	0.011 (0.001) *
Short-term Debt to Total Debt Ratio	0.473 (0.297)	0.051 (0.031)
Long term Debt to Total Asset ratio	0.503 (0.483)	0.054 (0.051)
Gross Profit Margin	-0.614 (0.169) *	-0.066 (0.020) *
Asset Turnover Ratio	-0.430 (0.199) **	-0.046 (0.023) **
Interest Expense to Debt Ratio	-8.679 (1.902) *	-0.938 (0.181) *
Net Working Capital to Net Sales	-0.015 (0.006) *	-0.002 (0.001) *
Cash to Net Sales Ratio	-1.339 (0.409) *	-0.145 (0.048) *
GDP Growth Rate	0.481(0.199) **	0.052 (0.022) **
Δ log (Money Supply, M2)	0.424 (0.213) **	0.046 (0.023) **
Δ log (Exchange rate)	3.497 (1.061) *	0.378 (0.123) *
Δ log (Forex Reserve)	0.492 (0.097) *	0.053 (0.011) *
Δ log (Balance of trade)	-2.843 (0.900) *	-0.307 (0.101) *
Δ log (Openness)	-0.703 (5.670)	-0.076 (0.613)
Industry dummy:		
Food	-0.033 (0.030)	-0.004 (0.003)
Chemicals, Chemical Products and Pharmaceuticals	-0.185 (0.036) *	-0.020 (0.004) *
Other Non-Metallic Mineral Products	-0.154 (0.024) *	-0.016 (0.002) *
Motor Vehicles	-0.479 (0.0719) *	-0.046 (0.007) *
Fuel & Energy	0.204 (0.094) **	0.025 (0.011) **

Information & Communication Services, others	0.169 (0.046) *	0.020 (0.006) *
Paper, Paperboard and Products	-0.821 (0.124) *	-0.072 (0.009) *
Electrical Machinery and Apparatus	-0.154 (0.067) **	-0.017 (0.006) *
Business cycle dummy:		
Recession	1.059 (0.498) **	0.111 (0.05) *
$\hat{\sigma}_a^2$	0.202	
Log-likelihood value	-3.196	

Note: ***Coefficient is significant at 10% level of significance, **significant at 5% level of significance, * significant at 1% level of significance. Standard errors are shown in parenthesis.

The classification results, presented in Table 4, compares the actual with the predicted outcome. It is found that the model correctly classified 96.5 % non-bankrupt firms while 73.8 % of the bankrupt firms. The low rate of true bankruptcy than non-bankruptcy classification is because of the disproportionate sample size of bankrupt versus non-bankrupt firms[11]. The false classification arises two types of errors[12]. The dynamic panel probit model produced 26.1 percent Type 1 and 3.4 percent Type 2 errors. The results are based on the in-sample forecast.

Table 4: Classification Results - Dynamic Panel Probit Model

	Predicted Group Membership			
	N = 3,460	**Non-bankrupt**	**Bankrupt**	**Total**
Original Count	Non-bankrupt	2,719	98	2,817
	Bankrupt	168	475	643
Percentage	Non-bankrupt	96.5	3.4	100.0
	Bankrupt	26.1	73.8	100.0

4.2. Results of the hybrid artificial neural network

The variables found significant in the dynamic panel fixed effect model is used to build an architecture of ANN as otherwise large networks heighten the risk of overfitting (Srivastava, Hinton, Krizhevsky, Sutskever, & Salakhutdinov, 2014). Therefore, only macroeconomic and financial variables remained in the model whereas none of the stock market variables found significant in predicting bankruptcy. Moreover, the study relies on the interpretation based on APE estimates of dynamic probit model

11 Only 17% of the total observations corresponds to bankrupt firms.

12 The prediction of bankrupt as non-bankrupt is Type 1 error and forecasting of non-bankrupt as bankrupt is known as Type 2 error in the context of the bankruptcy classification problem.

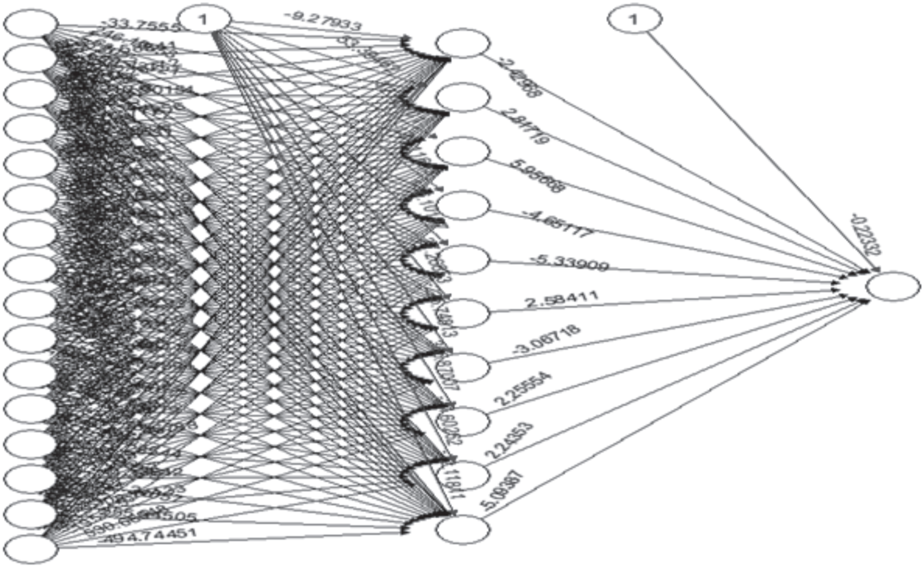

Figure 2: Architecture for Hybrid Artificial Neural Network

as hybrid ANN has difficulty in explaining estimated coefficients (Min & Lee, 2005).

Figure 2 represents the optimal network for the dataset employed for the study. Leftmost orbits are representing explanatory variables while middle orbits are the hidden neurons and finally, a rightmost singular orbit is representing the outcome variable. The numerals over each synapse (arrow connecting two orbits) are the weights derived by minimizing cross-entropy error. It is found that the optimal model consists of a single hidden layer with 11 hidden neurons.

Table 5 presents the classification accuracy corresponding to the hybrid ANN model. It is evident that the model attains 97.9 percent accuracy in classifying non-bankrupt companies, however, it gains 87.7 percent accuracy in classifying bankrupt companies. Alternatively, type 1 and type 2 error for hybrid-ANN are 12.2 percent 2.3 percent respectively.

Table 5: Classification Results - Hybrid-ANN Model

	Predicted Group Membership			
	N = 3,460	**Non-bankrupt**	**Bankrupt**	**Total**
Original Count	Non-bankrupt	2,759	58	2,817
	Bankrupt	79	564	643
Percentage	Non-bankrupt	97.7	2.0	100.0
	Bankrupt	12.2	87.7	100.0

4.3. Results of Merton's model

The probability of default is obtained from equation (10). The model gets an overall 73 percent accuracy for the in-sample forecast. The corresponding classification results of Merton's model are presented in Table 6. It is found that the model achieved 31.3% accuracy in classifying bankrupt firms while correctly classified 86.8 percent of non-. Relatively weak predictive ability and statistically insignificant coefficient of market variables bring some evidence of market inefficiency. It further suggests that stock market prices are default risk is not correctly priced. Merton's model produced 68.6 percent Type 1 and 13.1 percent Type 2 errors. It implies that primary parameters of Merton's model i.e. book value to total liabilities, the market value of the firm's asset, and the standard deviation of firm value do not contain much information pertaining to financial distress.

Table 6: Classification Results – Merton's Model

	Predicted Group Membership			
	N = 3806	**Non-bankrupt**	**Bankrupt**	**Total**
Original Count	Non-bankrupt	2,543	385	2,928
	Bankrupt	603	275	878
Percentage	Non-bankrupt	86.8	13.1	100.0
	Bankrupt	68.6	31.3	100.0

5. Model Diagnostics

5.1. Cross-validation analysis

Cross-validation is the statistical technique that allows comparing the learning methods by dividing the data set into two groups: first is to train the network and remaining is to validate (test) the fit. Zhang et al. (1999) used cross-validation to evaluate the appropriateness of the neural network for bankruptcy prediction.

This study uses *k*-fold cross-validation (where k=10 is taken because of its popularity). It implies that the total sample of 3,806 observations is divided into ten equal parts. Each class must appear once in both training and validating stages. The average misclassification rate (MR) is used to summarize the fit. However, PDs from the Merton's model are estimated by using distinct parameters for each firm. Therefore, following Charitou, Lambertides, and Trigeorgis (2008), regress binary dependent variable on PDs (from Merton's model) by means of logistic regression, cross-validation is then applied to this setting.

The results of the 10-fold cross-validation test are presented in Table 7. Moreover, an in-sample forecast is also summarized so that comparison can be made. It is found that hybrid-ANN outperforms other models regarding both in-sample and out-of-sample forecast estimates.

Table 7: Models Comparison: In-Sample and Out-of-Sample Forecasting Accuracy

Models	In-sample forecast accuracy	Out-of-sample accuracy
Dynamic Panel Model	92.3%	91%
Hybrid-ANN Model	96%	93.1%
Merton's Model	73.6%	74.04%

5.2. Receiver operating characteristics (ROC)

Receiver Operating Characteristics (ROC), for a two-class problem, allows visualizing trade-off between the rate at which model can accurately recognize positive cases vs the rate at which it mistakenly identifies negative.

The probability of defaults is derived from each model and use to classify firms into two classes (bankrupt versus non-bankrupt). The ROC curve is plotted by using actual and predicted classifications. The 45^0 line (random-guess line) divides the space into two portions. The curve farther from the diagonal represent a better fit as compared to the curve closer to the diagonal.

The area under the ROC curve (AUC) represents the rate of true classification. Figure 3 plots the area under the curve (AUC) of three bankruptcy prediction models employed in this study. Curves y_1, $y_{2,}$ and y_3 correspond to predicted values of dynamic probit, hybrid ANN, and Merton's model respectively. The comparison is performed using a non-parametric method to compare the AUC. It is found that the ROC curve of ANN has captured 98.6 percent area – the largest AUC among three models.

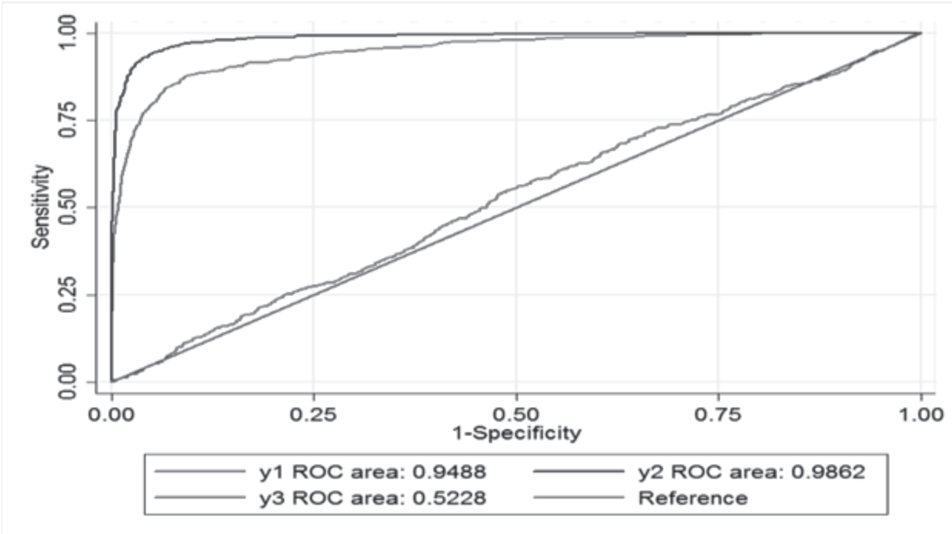

Figure 3: Receiver Operating Characteristic (ROC) curve

6. Conclusion

This study compares the predictive accuracy of three models i.e. dynamic panel probit, hybrid ANN, and Merton's model by employing three types of variables (financial ratios, stock-market based variables and macroeconomic indicators) based on the integration of finance-based definition with legal classification for listed non-financial companies in Pakistan. It extends the literature by, first, employing a combined definition for classifying financially distressed firms of Pakistan. Firms, in some cases, file bankruptcy in order to seek temporary protection without being a financially distressed provoking conflict of interest among stakeholders. A combination of finance-based definition with actual bankruptcy classification help mitigate such agency problems and can be used in simplifying formal procedures for tracing and protecting a business from ultimate bankruptcy. Second, the study offers the hybrid-ANN model, by combining dynamic panel probit model and ANN, which attains relatively higher predictive accuracy in terms of both in-sample and out-of-sample forecasts. The estimation of average partial effects in the context of the dynamic panel logit framework fills an important gap in bankruptcy prediction literature by explaining instantaneous changes in the probability of default in response to change in specific covariate while keeping other predictors constant.

It further contributes to the literature by introducing financial ratios, macroeconomic variables, and equity market variables simultaneously for estimating the probability of default. The financial ratios and macroeconomic variables only are found significant for classifying cases into two classes. This study further brings some evidence of mispricing of default risk as none of the stock-market based variables are found significant in explaining bankruptcy. It implies that the stock prices are not reflecting the true economic reality of the firms and raise concerns over stock market efficiency.

Given the dynamic nature of the characteristics of bankrupt/financially distressed firms over time, it is imperative for practitioners, regulators and academicians to test and enhance the bankruptcy prediction models. Therefore, research can further be enhanced by employing more sophisticated data science approaches like support vector machines, genetic programming and others. Similarly, other hybrid versions can also be tested. Moreover, the current study only includes the non-financial sector of Pakistan whereas the predictability of these models can also be tested in the context of the financial sector of Pakistan.

7. Implications of the study

The study brings useful implications for various stakeholders. Regulators can stipulate comprehensive bankruptcy law for better protection of businesses and can identify whether liquidation or reorganization suits best for the firms' financial condition by viewing shortlisted significant predictors in the event of bankruptcy. Similarly, financial institutions and creditors could get better insights into the financial performance of the firms before making lending decisions. Probability of default estimates are also useful for credit rating agencies and market participants base their parameter estimates on results reported in rating agency default studies. Testing default risk in asset pricing also requires proxy of default risk, therefore, accuracy in these estimates provide better insights. In sum, measurement of the probability of default is often considered as the first step in credit risk modelling.

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Appendix

Table 8: Review of Methodologies for Selected Bankruptcy Prediction Studies

Author	Variables	Technique	Data	Conclusion
Altman (1968)	Financial Ratios	Multiple Discriminant Analysis	78 firms over 1946-1965.	The model attained 94 % accuracy in terms of classification. The model develops well known Z-score (or Altman's Z-score).
Ohlson (1980)	Financial Ratios	Conditional logit model	105 bankrupt firms and 2058 non-bankrupt firms for 1970-1976	The binary logit model endorsed the predictive abilities of accounting ratios. However, it points out the limitation that market variables also contain some information.
Altman et al. (1994)	Financial Ratios	Artificial Neural Network, Linear Discriminant Analysis, logistic regression	1000 healthy, vulnerable and distressed industrial firms for 1982-1992.	Neural networks have potential predictive abilities but sensitive to overfitting. However, LDA gives better results.
Lee et al. (1996)	Financial Ratios	MDA, Iterative Dichotomiser 3 (ID3), Self-organizing feature map (SOFM).		
Hybrid models: MDA-assisted NN, ID3-assisted NN, SOFM-assisted NN	83 companies over 1979-1992. Matched sample in terms of size, capital size, number of employees, and age.	SOFM(MDA-assisted) NN performs better than other hybrid and individual models.		

Jo and Han (1996)	Financial Ratios	Discriminant Analysis, Neural Network, Case-based forecasting	31, 99, and 41 bankrupt firms and the same number of non-bankrupt firms in 1991,1992, and 1993 respectively	The predictive accuracy of an integrated model is far greater than that of individual models.
Shumway (2001)	Financial Ratios and Equity Market Variables	Simple Hazard model (similar to multi-period logit)	300 bankrupt firms observed over 1962-1992	The dynamic model has more predictive accuracy than that of static models.
Bharath and Shumway (2008)	Equity Market Variables	KMV-Merton model, Hazard model	1,449 firms over 1980-2003	The marginal benefits of KMV model come from its functional form. KMV-Merton probability is marginally useful default forecaster but not sufficient statistics. Moody's KMV is better than KMV-Merton.
Pindado et al. (2008)	Financial Ratios	Panel logit model	1,583 companies of US and 2,250 companies for G-7 countries over 1990-2002	The panel logit model has greater accuracy than the Altman Z-score model.
Agarwal and Taffler (2008)	Financial Ratios	Merton's model with two different approaches.		

UK-based Z-score model	2,006 non-financial firms over 1986-2001	Both the market-based approach (Merton model) and accounting-based models (Z-score) capture different aspects of bankruptcy. However, the accounting-based approach produces significant economic benefits over others.		
Tinoco et al. (2015)	Financial Ratios, Equity Market, and Macroeconomic Variables	Polytomous response (three-state) logit model	21964 non-financially distressed/ failed companies, 869 financially distressed, 385 failed companies for 2012	Model-based on the combination of accounting, market, and macroeconomic variables have higher predictive accuracy than the individual model at least for two lags.
Tinoco and Wilson (2013)	Financial Ratios, Equity Market, and Macroeconomic Variables	Logistic regression, Altman Z-score, comprehensive neural network (MLP)	23218 firm-years observations over 1980-2011	The combination of three types of variables in logistic regression outperforms the other models.
Khan (2018)	Financial Ratios	Multiple Discriminant Analysis (MDA) and the logistic regression model	40 financial institutions over 2009-2015	The logistic regression approach outperforms the multiple discriminant analysis (MDA) by achieving 81.5% predictive accuracy.

Table 9: Initial List of Variables

Type	Variables	Symbol
Market Variables		
Market	Adjusted Stock Price	M1
Market	Asset Return	M2
Market	Adjusted Sigma	M3
Market	Size	M4
Market	Market Capitalization to Total Debt Ratio	M5
Financial variables		
Liquidity	Current Ratio	A1
Liquidity	Quick Ratio	A2
Liquidity	Cash Ratio	A3
Liquidity	Working Capital to Total Asset	A4
Liquidity	Working Capital to Total Debt ratio	A5
Liquidity	Sales to Current Asset Ratio	A6
Liquidity	Current Liabilities to Total Liabilities Ratio	A7
Liquidity	Working Capital to Equity	A8
Liquidity	Current Asset to Total Asset	A9
Liquidity	Quick Asset to Total Asset Ratio	A10
Liquidity	Quick Asset to Inventory Ratio	A11
Liquidity	Cashflow to Short-term Debt ratio	A12
Liquidity	Cashflow to Total Asset Ratio	A13
Leverage	Debt to Equity Ratio	A14
Leverage	Long-term Debt to Equity Ratio	A15
Leverage	Cash Coverage Ratio	A16
Leverage	Funded Capital Ratio	A17
Leverage	Total Debt to Total Asset Ratio	A18
Leverage	Short-term Debt to Total Debt Ratio	A19
Leverage	Long-term Debt to Total Asset Ratio	A20
Leverage	Interest Coverage Ratio	A21
Leverage	Long-term Debt to Equity Ratio	A22
Leverage	Short-term Debt to Equity Ratio	A23
Leverage	Total Debt to Equity Ratio	A24

Profitability	Earnings Before Taxes to Revenues Ratio	A25
Profitability	Net Profit Margin	A26
Profitability	Return on Equity	A27
Profitability	Return on Asset	A28
Profitability	Return on Capital	A29
Profitability	Operating Profit Margin	A30
Profitability	Operating Income Margin	A31
Profitability	Gross Profit Margin	A32
Efficiency	Inventory Turnover Ratio	A33
Efficiency	Asset Turnover Ratio	A34
Efficiency	Cashflow to Total Debt Ratio (Debt Coverage Ratio)	A35
Efficiency	Interest Expense to Debt Ratio	A36
Efficiency	Tax Rate Percentage	A37
Activity Ratio	Net Working Capital to Net Sales	A38
Activity Ratio	Tangible Fixed Assets Turnover ratio :(net sales/fixed assets)	A40
Activity Ratio	Long-term Debt Turnover Ratio	A41
Activity Ratio	Quick Asset to Net Sales Ratio	A42
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Macroeconomic variables		
Macroeconomic	Terms of Trade	E1
Macroeconomic	GDP Growth	E2
Macroeconomic	Money Supply (M2)	E3
Macroeconomic	GDP Deflator	E4
Macroeconomic	Current Account Balance	E5
Macroeconomic	Exchange Rate	E6
Macroeconomic	Forex Reserves	E7
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Macroeconomic	Trade Openness	E10
Macroeconomic	Interest Spread	E11
Macroeconomic	Industrial Production	E12
Macroeconomic	Consumer Price Index	E13

Table 10: Variance Inflating Factor[13] and Tolerance Value

Variable	VIF	1/VIF	Variable	VIF	1/VIF	Variable	VIF	1/VIF
A4	614611.19	0.00	A30	289.15	0.00	A13	1.94	0.52
A18	614016.63	0.00	A20	276.69	0.00	A10	1.92	0.52
E13	305210.56	0.00	A1	216.58	0.00	A33	1.91	0.52
E14	166178.64	0.00	A5	211.03	0.00	A11	1.86	0.54
A24	137323.98	0.00	A36	131.02	0.01	A3	1.76	0.57
A14	124266.91	0.00	A9	87.56	0.01	A27	1.74	0.57
E6	58676.33	0.00	A39	77.59	0.01	A17	1.72	0.58
E2	6762.91	0.00	A32	67.35	0.01	A35	1.72	0.58
A15	4311.32	0.00	A21	34.21	0.03	M2	1.25	0.80
A23	3659.63	0.00	A16	34.18	0.03	A28	1.20	0.83
E7	3403.06	0.00	A38	16.67	0.06	M4	1.18	0.85
E12	2643.82	0.00	A43	10.38	0.10	M1	1.17	0.86
E8	1951.18	0.00	A12	7.37	0.14	A40	1.14	0.88
E11	1582.10	0.00	A42	4.55	0.22	M3	1.05	0.95
E4	1561.54	0.00	A8	3.57	0.28	A29	1.03	0.97
E3	1548.09	0.00	A2	3.25	0.31	A41	1.02	0.98
A25	1058.78	0.00	A34	3.24	0.31	A37	1.01	0.99
A31	729.35	0.00	A6	2.60	0.38			
A26	584.82	0.00	A7	2.58	0.39			
E1	324.26	0.00	M5	2.44	0.41			

13 VIF is calculated as:

$$VIF_i = \frac{1}{1 - R_i^2}$$

where Ri2 is the coefficient of determination when ith independent variable is regressed over others. Tolerance value is simply the reciprocal of VIF.